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THEORETICAL BASIS OF THE PROBABILISTIC HYDROLOGICAL MODEL MARCS (MARKOV CHAIN SYSTEM).

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An advanced of frequency analysis (AFA) method adopts the theory of stochastic systems to be applied in hydrological engineering.

- **Introduction:** The AFA method has been suggested more then 20 years, however, the full description of this approach is still unpublished in English. The paper formulated "step-by-step" the theory and assumptions of the AFA method behind a code of probabilistic hydrological model MARCS (MARkov Chain System).
- **Model theory:** MARCS model is based on the theory of stochastic systems, specifically, the Fokker-Plank-Kolmogorov equation (FPK), which is simplified to a system for three statistical moments (Fig. 1). The time series of multi-year runoff are considered as realization of random process Markov chain type.
- **Results: N**ew parametrization scheme of the probabilistic hydrological model MARCS is implemented to be able simulation the location parameter of the Pearson Type III distribution previously assumed to be constant (Shevnina et al., 2017; Kovalenko et al., 2010). Fig. 2 shows the new parametrization scheme of the MARCS model in a nut-shell. This scheme is used in the model set-up in (Shevnina E., Pilli-Sihvola, K., Haavisto, R. and Vihma, T., 2017: probabilistic hydrological scenarios for Scandinavia and Northern Russia: the perspectives in economical applications, IASC meeting Umea, Sweden).

$$m_{n-1}b_0 n + m_n[b_1(n+1)-a] + m_{n+1}[(n+2)b_2+1]=0$$

Core Model: Linear filter with stochastic component

$$a=\frac{G_{\tilde{c}\tilde{N}}+2\bar{N}}{2\bar{c}+G_{\tilde{c}}} \quad b_0=-\frac{G_{\tilde{N}}}{2\bar{c}+G_{\tilde{c}}} \quad b_1=\frac{2G_{\tilde{c}\tilde{N}}}{2\bar{c}+G_{\tilde{c}}} \quad b_2=-\frac{G_{\tilde{c}}}{2\bar{c}+G_{\tilde{c}}}$$

$$-k(\bar{c}-0.5kG_{\tilde{c}})m_k + k\bar{N}m_{k-1} - k(k-0.5)G_{\tilde{c}\tilde{N}}m_{k-1} + 0.5k(k-1)G_{\tilde{N}}m_{k-2}=0$$

Regime type: quasi-stationary period

$$m_1(2b_2+1)-a+b_1=0$$
$$(3b_2+1)m_2+(2b_1-a)m_1+b_0=0$$
$$(4b_2+1)m_3+(3b_1-a)m_2+2b_0m_1=0$$
$$(5b_2+1)m_4+(4b_1-a)m_3+3b_0m_2=0$$

Distribution type: the Pearson Type III

$G_{\tilde{c}} << \bar{c}$ $\quad b_2=-G_{\tilde{c}}/(2\bar{c}+G_{\tilde{c}})\approx 0 \quad (4b_2+1)\approx 1 \quad (2b_2+1)\approx 1$

$$-a+b_1=-m_1$$
$$b_0+2m_1b_1-am_1=-m_2$$
$$2m_1b_0+3m_2b_1-am_2=-m_3$$

m_1 ==> *Norm*, $\bar{Q}=m_1$

m_1, m_2 ==> *Coefficient of variation*, $CV=\sqrt{(m_2-m_1^2)}/m_1$

m_1, m_2, m_3 ==> *Coefficient of skewness*, $CS=(m_3-3m_2m_1+2m_1^3)/(CV^3 m_1^3)$

$$\bar{c}_r=\bar{N}_r/(a_r-b_{1r}/2)$$
$$G_{\tilde{N}r}=-2b_{0r}\bar{N}_r/(a_r-b_{1r}/2)$$
$$G_{\tilde{c}\tilde{N}r}=b_{1r}\bar{N}_r/(a_r-b_{1r}/2)$$

$$m_k=1/n\sum_{i=1}^{n}Q_i^k \qquad \bar{N}=1/n\sum_{1}^{n}PRE$$

Statistical estimator for runoff: initial moments

... for climate: mean values

$\bar{c}_r=\bar{c}_{pr}, G_{\tilde{N}r}=G_{\tilde{N}pr}, G_{\tilde{c}\tilde{N}r}=G_{\tilde{c}\tilde{N}pr}$

Options: No changes on the variability of catchment physiography (VCP), No changes on the variability of precipitation (VP), No changes on the mutual variability of VCP and VP.

Figure 2. The MARCS model set-up for Scandinavia in a nut-shell.

Simple "Black Box" Model, WMO: 2009.

$$a_n(t)\frac{d^nQ}{dt^n}+a_{n-1}(t)\frac{d^{n-1}Q}{dt^{n-1}}+...+a_1(t)\frac{dQ}{dt}+a_0(t)Q=$$
$$=b_n(t)\frac{d^nP}{dt^n}+b_{n-1}(t)\frac{d^{n-1}P}{dt^{n-1}}+...+b_1(t)\frac{dP}{dt}+b_0(t)P$$

where, Q is model output (runoff), P is model input (precipitation), an and bn are parameters connceted to catchment physiograpy and meteology.

WMO, 2009: WMO No.168: the Guide to Hydrological Practices. Volume II: Management of Water Resources and Application of Hydrological Practices, Chapter 6: Modelling of hydrological system. web: http://www.whycos.org/hwrp/guide/

"Linear filter with stochastic component" Model, Pugachev et al.: 1974.

$$\frac{dY}{dt}=f(Y,t)+g(Y,t)\xi(t)+h(Y,t)\eta(t)$$

where, is Y model output, $\xi(t)$ and $\eta(t)$ are Gaussian noise signals with zero means and intensities D_ξ and D_η ; the signals are mutually correlated and their correlation function:

$$K_{\xi\eta}(\tau)=E(\xi(t)\eta(t+\tau))=D_{\xi\eta}\delta(\tau)$$

Pugachev V.S., Kazakov I.E., Yevlanov L.G., 1974: Basics of statistical theory of automatic systems. Chapter 9: Stochastic systems. Moscow, Mashinostroenie.

"Pearson Distributions" Model (System), Pearson: 1895.

$$\frac{dp(Q)}{dQ}=\frac{Q-a}{b_0+b_1Q+b_2Q^2}p(Q)$$

where: a, b_0, b_1 and b_2 are parameters of Pearson System.

Pearson K., 1895. Memoir on Skew Variation in Homogeneous Material, Philosophical Transactions of the Royal Society, A186, pp. 323-414.

Statistical Estimation Methods for Engineering, see review in Van Gelder, 2004.

Van Gelder, P.H.A.J.M., Statistical Estimation Methods in Hydrological Engineering, Proc. Intern. Semin. "Analysis and Stochastic Modeling of Extreme Runoff in Eurasian Rivers Under Conditions of Climate Change," Irkutsk: Publishing House of the Institute of Geography SB RAS, 2004, pp. 11–57.

$$m_{n-1}b_0 n + m_n[b_1(n+1)-a] + m_{n+1}[(n+2)b_2+1]=0$$

Kovalenko V.V, 1993. Modelling of hydrological processes. Leningrad, Hydrometizdat.

Figure 1. The MARCS model basic theory in a nut-shell.

- **Discussions:** the parameterization scheme of the MARCS model was developed to simulate three statistical moments for the projected period based on climate projections. However, the model is still adopted to be forced by the projections coming from global climate models. The global model have coarse resolutions and this circumstance is crucial for the physically-based hydrological models.
- **Conclusion:** The probabilistic hydrological MARCS model should be tested for a sensitivity to the climate forcing coming from global and regional scales climate models.
- **Code availability:** the code of current version of the probabilistic hydrological model MARCS can downloaded from (github.com/ElenaShe000/MARCS) and used under the GNU3.0.
- **Acknowledgements**: the study is supported by the Academy of Finland (contract 283101)
- **References:** Kovalenko et al. 2010: Guideline to estimate a multi-year runoff regime under non-steady climate to design hydraulic contractions, RSHU, Saint-Petersburg. (In Russian). Shevnina at al., 2017: Assessment of extreme flood events in a changing climate for a long-term planning of socio-economic infrastructure in the Russian Arctic, Hydrol. Earth Syst. Sci., 21, 2559-2578.